2 Biomechanics of Tilted Implants

Appendix

This appendix reviews the key mechanical concepts that arise in chapter 2, including force, torque (moment), stress, strain, stress-strain relationships, structural and material properties, and material/structural failure.

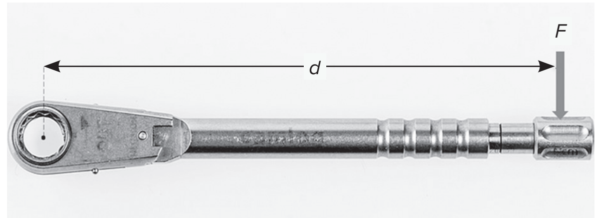
Force

The common meaning of *force* is a push or pull. For example, when chewing food, the maxillary and mandibular teeth generally come together to crush a bolus of food with a compressive (pushing) force. A good example of a tensile (pulling) force arises when muscles activate during chewing. For example, the masseter muscle, which runs between the zygomatic arch and the angle/lateral surface of the ramus of the mandible, contracts to exert a tensile force on the mandible, pulling it toward the maxilla.

In addition to the type of force (eg, tensile, compressive, shear, etc), there are two other defining characteristics of a force: magnitude and direction. Force is a vector quantity. For example, when a patient bites on a food particle caught between the crown of a dental implant in the mandible and the opposing crown of a natural tooth in the maxilla, the force on the mandibular crown of the implant might have a magnitude of 100 N, and its direction can be represented by a downward-pointing arrow acting on the crown. Likewise, one can envision an upward-pointing arrow (with a magnitude of 100 N) acting on the maxillary crown of the opposing tooth—an example of Newton's Third Law of Motion (action and reaction).

Confusion sometimes arises between the terms *force* and *pressure*. Force is measured in units such as pounds (lb) or Newtons (N). Pressure, which is actually a type of stress, is measured in units of force per unit area (F/A), with units such as lb/in^2 (psi) or N/m^2 (Pascal; Pa). For example, when a patient bites on a piece of hard candy with a force of 100 N, suppose the 100 N force is distributed over a very small area of contact (1 mm^2) where the candy touches the tooth's enamel surface. Here it's correct to state that the bite force on the candy is 100 N, while the contact pressure between the tooth and candy is 100 N/1 mm^2, which can also be expressed as 100 N/10^{-6} m^2 = 100 × 10^6 Pa = 100 MPa. If the area of contact were even smaller, say 0.5 mm^2, then the contact pressure would be 200 MPa. Thus, with the same bite force of 100 N, one can generate different bite pressures (stresses) during chewing, depending on the contact area over which the force acts. So, when describing the value of a bite force, it's appropriate to use force units, not pressure or stress units.

There is also sometimes confusion between *force* and *mass*. For example, a typical orthodontic "force" to move a tooth might be in the realm of 30 g (0.03 kg), which is strictly a contradiction in terms because a gram is a unit of mass, not force. However, a 30-g mass is equivalent to a force whose magnitude can be determined using the well-known equation $F = ma$, where F is force, m is mass, and a is acceleration. In this orthodontic example, the value of acceleration a would be g, the acceleration due to gravity at sea level, 9.81 m/s^2. (Here, the equation $F = ma$ can take the form $W = mg$, where W represents weight.) So, 30 g (0.03 kg) is equivalent to a force of 0.29 N.

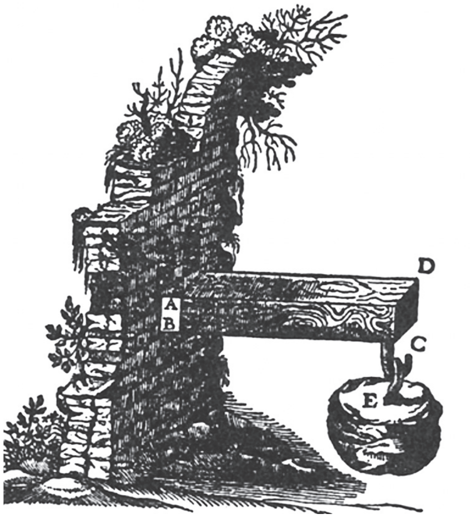


APP FIG 2-1 A simple torque wrench for dental implant applications.

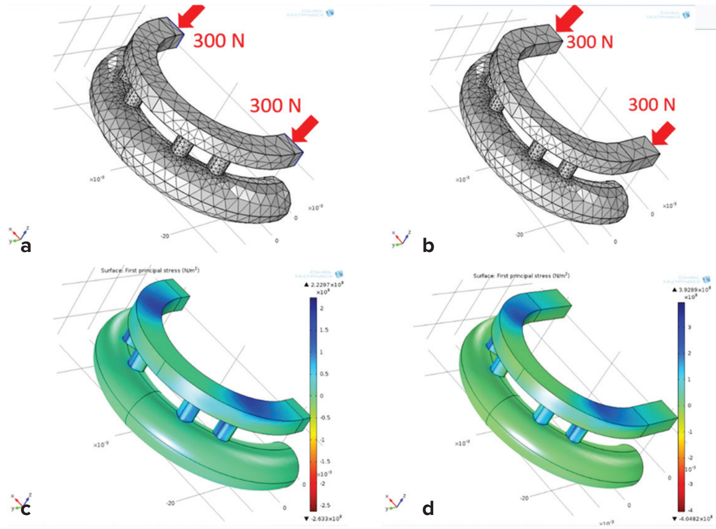
Moment (Torque)

A moment (or torque) describes the tendency of a force to produce rotation around a point or an axis. Common examples of the use of torque in dental implantology include inserting a screw-shaped implant into an osteotomy site at some prescribed insertion torque value or tightening an abutment screw. Moment/torque is reported as force times distance, with units such as Ncm. This can be understood from studying App Fig 2-1, showing a type of dental torque wrench. One uses the wrench by applying a force F perpendicular to the handle at some distance d from the center of the implant or abutment screw. The resulting torque at the implant (also called a *moment*, abbreviated as M) would be computed as $M = Fd$. So, if a 5-N force were to be applied perpendicular to the handle of the wrench 7 cm from the centerline of the implant or abutment screw, the magnitude of the applied moment (torque) at the implant or screw would be 5 N × 7 cm = 35 Ncm.

In mechanics, a moment or torque is also a vector quantity, and in this example, the 35-Ncm torque would have a direction defined by an arrow pointing into the page. Another way to convey this directionality is to say that the torque in this example is causing a clockwise rotation of the implant or abutment screw.



APP FIG 2-2 A sketch of a wooden cantilever beam from the early work of Galileo.



APP FIG 2-3 Stress analyses of 17-mm and 25-mm cantilevers that are part of full-arch prostheses loaded bilaterally with 300 N. *(a and b)* Model geometry. *(c and d)* Distributions of tensile stress throughout the model.

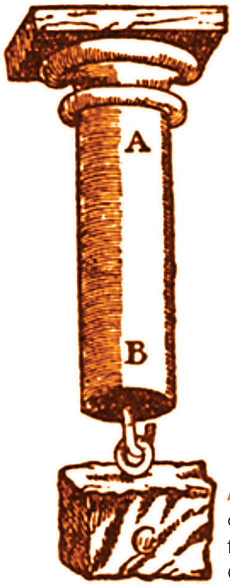
Another example of a moment (torque) comes from the time of Galileo.[1] He considered the case of a wooden beam loaded by a weight *E* hung from the beam's end (App Fig 2-2), ie, a beam loaded as a cantilever. For discussion purposes, the weight of the beam itself will be neglected. If the length of the beam is *d* and the weight *E* exerts a force *F* at the end of the beam, then the moment (or torque) at the location where the beam meets the wall is equal to *Fd*. This is also called a *bending moment*.

This analysis is relevant to dental implantology because this cantilever beam is analogous to the cantilevers that sometimes exist in a full-arch prosthesis supported by several implants, as depicted in a finite element (FE) computer model (App Fig 2-3). The FE model illustrates a *U*-shaped, full-arch titanium prosthesis supported by four titanium implants installed in a simplified semicircular mandible made of cortical bone. The FE model considers two different cantilever lengths, 17 mm and 25 mm, with the cross-sectional shape of the titanium prosthesis always the same. Each prosthesis is loaded bilaterally by 300-N forces acting downward (negative *z* direction) at the distal end of each cantilever. Using the same approach as with Galileo's wooden beam, each distal part of the prosthesis can be approximated as a cantilever beam with one end fixed at the distal implant. It follows that the moment acting on the prosthesis at the location of the most distal implant is *Fd*, where *d* is the cantilever length and *F* is 300 N. So, for the 17-mm cantilever, the moment would be 300 N × 17 mm = 510 Ncm, while for the 25-mm cantilever, the moment is 750 Ncm. Therefore, the longer the cantilever, the greater the moment is at the point where the prosthesis is supported at the distal implant.

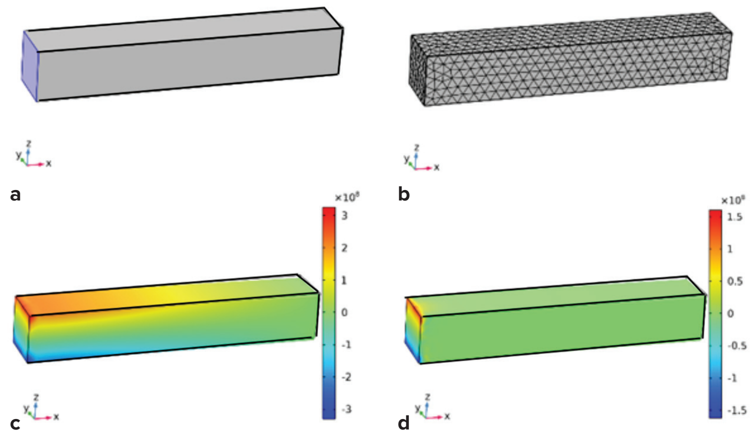
The relevance of this analysis involves the concept of stress, which is covered in the next section, but for now we can point out that the stress in a cantilever beam depends on the value of the bending moment at the location of interest, so the 25-mm cantilever will undergo greater stresses than the 17-mm cantilever. Indeed, FE results show that the stress at the top surface of the 25-mm cantilever is roughly twice as large as it is for the 17-mm cantilever at the location of the distal implants (note the color scale for stress and the dark blue color indicating stresses of a few hundred MPa on the top surface of the beams in App Fig 2-3). As to the physical meaning of stress, see the next section.

Stress

In mechanics, the concept of stress quantifies what happens inside the body of a material when it is subjected to external loads such as forces, moment, prescribed deformations, or some combination of all three. A simple, everyday example of stress arises when one squeezes a soft, solid rubber ball in one's hand. The



APP FIG 2-4 Example of a column in tension, from the early work of Galileo.



APP FIG 2-5 An FE analysis of a titanium cantilever beam with the left end fixed and the right end loaded by 100 N in the negative *z* direction. *(a and b)* Geometry and mesh of the model. *(c and d)* Distributions of stress along the longitudinal and lateral directions, respectively.

forces from the hand cause the ball to deform, and as this happens, internal forces develop inside the ball, a fact that can be deduced from the fact that the ball springs back to its original shape after the hand stops squeezing it. The internally generated forces in the ball tend to restore it to its original shape.

An example of stress along just one direction—uniaxial stress—is depicted in an early illustration by Galileo (App Fig 2-4). A uniform column has a weight *C* applied to the end near point *B* while the top of the column near point *A* is held fixed. Suppose the force (weight) from the attached block *C* at the lower end of the column is a downwardly directed force *F*. Then, if the cross-sectional area of the column is the same value *A* everywhere along the column's length, the stress in the column is *F/A* and would be termed a *tensile stress*, ie, a stress tending to elongate the column. A similar example would be hanging a small weight from a rubber band. If the direction of *F* were reversed, the magnitude of the stress in the column would still be *F/A*, but it would be assigned a negative sign to indicate that the stress is compressive. The dimensions of stress are force per unit area.

Additional important features of stress include the fact that a stress is not always uniaxial, nor does a stress always have the same value (magnitude) or directionality from point to point in a material. Depending on a number of factors (such as the geometry of a body and how it is loaded, etc), the stress state throughout the body will sometimes be multiaxial and spatially nonuniform. *Stress state* refers to the stress condition at a given point in the body, and *multiaxial* means that stresses at a point may be acting in more than one direction because more than one stress component exists. Unfortunately, because stress is more complicated than a scalar quantity, such as length, or a vector quantity, such as force, a complete explanation is beyond the scope of this appendix. However, the following example illustrates some key features to keep in mind.

Revisiting Galileo's cantilever beam in App Fig 2-2 and the FE model of the cantilever region of a dental prosthesis in App Fig 2-5, consider a cantilever beam from a prosthesis that measures 25 mm × 4 mm × 4 mm in the *x*, *y*, and *z* axes, respectively. Suppose the beam is made of pure titanium and loaded by a downward force *F* of 100 N at the end of the right face of the beam, while the left face of the beam (shaded blue in App Fig 2-5a) is held fixed. The beam is shown with the FE mesh in App Fig 2-5b. Example results from the stress analysis are shown in App Fig 2-5c and 2-5d.

App Fig 2-5c is a view of the surfaces of the beam with color-coding displaying the stress component acting in the *x* direction, longitudinal stresses parallel to the length of the beam. The regions with reddish-yellow colors on the top surface of the beam illustrate tensile (positive) stresses, which are greatest in magnitude at the left (fixed) end of the beam. Similarly, the dark blue and light blue regions at the bottom half of the beam illustrate longitudinal compressive (negative) stresses, which are greater in magnitude at the fixed end of the beam than at the free end. This finding also illustrates that the tensile and compressive stresses in this beam are greatest where the bending moment is greatest—at the fixed end of the beam. These results also show that the stress is spatially nonuniform.

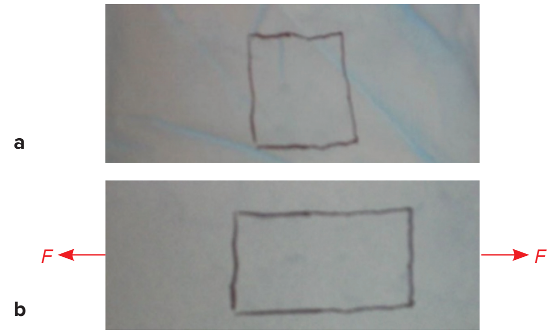
App Fig 2-5d illustrates that at a given point in a body, stress might be acting in more than one direction (ie, the stress state is multiaxial). App Fig 2-5d displays the stress component that acts along the y-axis, and App Fig 2-5c shows the stress component that acts along the x-axis. So, we find that at various locations in the beam, stresses are acting along both the x and y axes at the same time. Though not shown here, there is also a stress component for this beam in the z-axis.

Strain

In mechanics, strain is a measure of deformation. The formal terms used to describe strain in the material of a body are similar to those used to describe stress. For example, as with stress state, the strain state at a given point in a body depends on factors including how the body is loaded and the body's geometry. The same questions also arise about whether strain state is uniaxial or multiaxial and whether the strain distribution in the body of a material is spatially uniform.

Consider an initial example of strain that's easy to visualize with the naked eye. Take a small patch of a thin rubber examination glove, draw a small square on it with a marker, and stretch the patch with a tensile force F along the horizontal direction (App Fig 2-6). Stretching a common rubber band will also work for this exercise. Before the rubber is loaded, the outline is approximately square (see App Fig 2-6a), but after loading, the square becomes rectangular (see App Fig 2-6b). That is, in the horizontal direction the square has elongated, but in the vertical direction the square has narrowed.

To quantify what's happening, a common definition of *strain*, e, is a change in length divided by original length, as in the equation $e = (L_f - L_o)/L_o$, where L_f and L_o are final length and original length, respectively. If one used a ruler to take measurements from the images in App Fig 2-6, one could then use the strain equation to calculate a positive value of strain (equal to about 0.8) in the horizontal direction, and a negative value of strain (equal to about −0.125) in the vertical direction.



APP FIG 2-6 *(a and b)* Images during a uniaxial tensile test of a patch of rubbery material from a dental examination glove. The ink square in *a* has deformed into a rectangle after a tensile force *F* load is applied in *b*.

These fractions could then be converted to percentages by multiplying each fraction by 100.

An interesting result is that even though the applied force F causes uniaxial tension (with stress only along the x-axis), the strain state is multiaxial, with a tensile strain occurring along the x-axis and a compressive strain occurring along the y-axis. Not shown is the fact that there's also a compressive strain (about 0.125) along the z-axis. Meanwhile, the strain state is spatially uniform over the sample of rubber. Thus, the state of strain in this body is three-dimensional even though the stress state is one-dimensional—a fact that leads into the following section on the idea of stress-strain relationships and material properties.

Stress-Strain Relationships and Material Properties

If one considers a common coiled spring, its idealized behavior is described by the well-known expression $F = kx$, where F is the force exerted on the spring, x is the amount of stretch (or compression) of the spring, and k is the spring constant. This equation denotes a linear relationship between force and extension: To double the stretch of the spring, twice the force is required. This also implies elastic behavior, where if the spring is stretched and then unloaded, it would return to its original length. And while the spring constant k in this equation is actually more of a structural property, with a value depending not only on the material of the spring but also on the shape/size/design of the spring, it is analogous to a material property that arises in the stress-strain relationship.

The simplest stress-strain relationship for a material follows the same model as the ideal spring equation. That is, for many common man-made materials (eg, titanium, acrylic, zirconia), and for at least a few biologic materials (eg, dense cortical bone, dentin, enamel), experiments show that stress and strain are linearly proportional to one another. The simplest stress-strain equations are for those materials with linear elasticity, in which the material's behavior under loading and unloading is the same and in which stress is linearly proportional to strain. Moreover, the equations are most simple when the material in question is isotropic, with the material properties being the same throughout. As materials become more complex, as in an anisotropic material, the stress-strain relationships become more complicated.

Starting with the simplest material model of a linear, isotropic, elastic solid, the equations representing the stress-strain relationship involve various constants—material properties, such as moduli of elasticity (eg, Young's elastic modulus, bulk modulus, shear modulus, etc), as well as Poisson's ratio. These material properties are analogous to the spring constant k in the ideal spring equation. The significance of knowing the appropriate stress-strain relationship for a given material is that the equation allows for prediction of the strains in a material when the stresses are known, and vice versa.

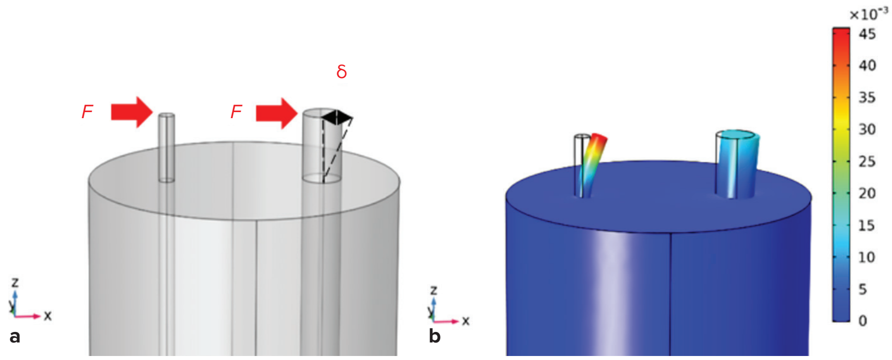
The simplest example of a stress-strain relationship is an equation that applies (only) in the case of an isotropic, linearly elastic material under a uniaxial stress σ, namely $\sigma = E\varepsilon$, where E is Young's elastic modulus of the material in question and ε is strain in the same direction as the stress. For a numerical example of how this equation works, consider the act of stretching a common rubber band as per the rubber glove example in App Fig 2-6. Suppose the equation applies and the value of E for the rubber band is 1 MPa. If a uniaxial tensile force of 4 N is applied to a rubber band with a cross-sectional area of about 6 mm^2, the uniaxial stress is 0.66 MPa. What is the predicted strain in the rubber band? Calculation shows that the strain ε is 0.66 MPa/1 MPa = 0.66, or 66%.

We could take this example one step further. Suppose we know another constant in the stress-strain relationship of rubber, namely the Poisson's ratio, which is defined as the negative of the ratio of lateral to longitudinal strain in a condition of uniaxial stress. If Poisson's ratio is known to be 0.45 for rubber and the longitudinal strain was 0.66 in our uniaxial stress test above, the lateral strain is predicted to be about 0.3 ($0.66 \times 0.45 = 0.297$).

Generally, when performing stress-strain analyses for real, complex prostheses, implants, bone, teeth, periodontal ligament, etc, one will be confronted with much more complicated geometries, as well as significant uncertainties about the actual material properties of the tissues involved. While geometric complexity can be solved by using advanced methods such as FE analysis, many biologic materials are anisotropic, having different material properties in different directions; viscoelastic, having time-dependent material properties; nonlinear in terms of the stress-strain relationship; biphasic, meaning they are comprised of fluid and solid phases; and also nonuniform, having different chemical or physical makeup from place to place on the micro scale. Unfortunately then, given current knowledge of the material properties of biologic materials, it is not often possible to account for the true complexity of the materials and systems in oral implantology. An important caveat is that in any mechanical analysis in engineering, one must start somewhere, and the analyst must strike an appropriate balance between several factors: *(1)* the (usually highly) idealized material models and assumptions employed in the analysis, *(2)* the real properties of the materials being studied, and *(3)* the overall goals of the analysis at hand. A pertinent quotation attributed to the statistician George E. P. Box states, "All models are wrong, but some are useful"—meaning that even if a model cannot perfectly replicate reality, it can at least serve as a starting point or perhaps be close enough to give some insight.

Material Versus Structural Properties

Confusion sometimes arises between the mechanical properties of a structure and the mechanical properties of the material making up the structure. For clarity, a *structure* is "any object that must support or transmit loads."[2] A *material* is what comprises the structure. Obviously, when a real structure supports or transmits loads, eg, a zygomatic implant that's loaded in bone, the structure's material also functions in the support or transmission of loads. But one can have a situation where a strong material is part of a structure that ends up being inadequate to the task at hand. For example, while most of the metals used in implant dentistry are themselves "strong," it is nevertheless possible to have a "strong" metal configured in such a way that it is actually "weak"—at least compared to requirements of the specific structural situation.



APP FIG 2-7 *(a)* Lateral stiffness tests via an FE model of two idealized zygomatic implants represented as uniform solid cylinders in dense bone. The smaller implant's diameter is 2 mm, and the larger implant's diameter is 5 mm. *(b)* The *x*-displacement of the implants is shown after a 100-N lateral force is applied (displacement magnified 50×). The color scale is in millimeters.

To make this point with a simple and admittedly contrived example, let's consider the lateral stiffness in bone of two potential designs of a titanium zygomatic implant. We will assert that the lateral stiffness is in fact a structural property, defined as the constant K in the equation $F = K\delta$, where F is the lateral force applied at the top of the implant and δ is the resulting lateral displacement of the top of the implant (App Fig 2-7a). The fact that K is a structural property (and not a material property) is evident from an FE model. Suppose each zygomatic implant is a 50-mm long cylinder made of the same grade of titanium, but one implant is 2 mm in diameter and the other is 5 mm in diameter. Suppose 43 mm of each implant is well integrated in a cylinder of dense bone, leaving 8 mm of each implant protruding out of the bone. A lateral force of 100 N acts in the x-direction on the top of each implant. How does the lateral stiffness of the two implants compare?

App Fig 2-7b illustrates the answer: Under a 100-N lateral force in the x-direction, the smaller-diameter implant tilts (displaces) in the x-direction about 45 µm, and the larger-diameter implant tilts about 14 µm. Thus, the K values for the smaller- and larger-diameter implants are computed from $K = F/\delta$, that is, 2.22 N/micron and 7.14 N/micron, respectively. The larger-diameter implant has the greatest lateral stiffness. Note that this is true even though the material properties (eg, elastic modulus, Poisson's ratio) of the two implants are identical. In other words, the behavior of the implant under loading is appreciably determined by the diameter (structure) of the implant, so stiffness here is a structural property. Certainly, it is true that the material properties of the implant and surrounding bone also contribute to the lateral stiffness of the implant. If the elastic modulus of the implant and bone were decreased, the lateral stiffness of both implants would decrease. But the structure's size and shape are key determinants of the stiffness.

This is clinically significant because measuring the lateral stiffness of an implant is one way to quantify the implant's stability, ie, its ability to resist a lateral force. If the lateral stiffness of an implant is too weak, the implant may displace too much under functional loading and fail to adequately support a loaded prosthesis. The clinician must understand that lateral stiffness depends not only on the material that makes up a structure but also the size and shape of the structure. The reader will find an article by Monje et al on the relationship between mechanical and biologic implant stability to be pertinent to this topic.[3]

Structural and Material Failure

As noted previously, structural properties and material properties are different, although together they determine the behavior of a structure in a given mechanical situation, including during failure. For example, the displacement (deflection) of the top of the smaller-diameter implant versus the larger one from the last section could have been approximated using an equation from cantilever beam theory (which actually only considers a beam fixed at one end and not a beam embedded in bone):

$$\delta = \frac{FL^3}{3EI}$$

where δ is deflection, F is force, L is the length of the "beam," E is Young's elastic modulus of the material in the beam, and I is the second moment of inertia of the beam's cross section. For a beam with a circular cross section with diameter d:

$$I = \frac{\pi d^4}{3EI}$$

This equation indicates that if *d* is smaller, *I* will also be smaller, which in turn means that the deflection δ will be larger, all other factors remaining the same. This comports with the results obtained with the FE model—the 2-mm-diameter implant deflects more than the 5-mm-diameter implant. But this equation also reveals that the deflection depends on the elastic modulus of the beam *E*, which is a material property. So, rearranging the equation so it resembles the form used in the last section ($F = K\delta$), we obtain:

$$F = \frac{3EI}{I^3}\delta$$

The term $3EI/L^3$ can be thought of as *K*, the structural stiffness in the equation $F = K\delta$. This shows explicitly that *K* depends on both geometric *(I)* and material *(E)* properties.

When it comes to the failure of a structure, this is usually accompanied by a failure of a material in the structure. Returning to the prostheses structures analyzed in App Fig 2-3, for example, at least two questions arise: *(1)* What types of failure are possible? *(2)* What types of failure are likely?

Typically, possible failure modes of common engineering materials such as titanium and other metals include yielding, single-cycle overload, and fatigue (which is failure after many loading cycles, eg, 107 cycles). Yielding means that the stress in the metal has exceeded the yield strength so that a permanent deformation (eg, a bend) of the metal has occurred. Single-cycle overload is captured in the ultimate tensile strength, which is the tensile stress that causes complete fracture in a uniaxial tensile test. Values for these strengths are commonly listed in textbooks, handbooks, and websites.

For an example application of these data, consider the cantilever portions of the prostheses in App Fig 2-3. The FE results indicate that tensile stresses are concentrated at the top surfaces of both the 17- and 25-mm cantilever portions of the prostheses, with peak stresses of about 222 MPa and 394 MPa, respectively. Commercial-purity, cold-worked titanium has a yield strength of about 485 MPa and an ultimate tensile strength of about 760 MPa. Thus, both values significantly exceed the peak tensile stress in either beam, suggesting that yielding or overload will not occur in one cycle of 300-N loading. On the other hand, if failure by fatigue is considered, the 394-MPa peak tensile stress of the 25-mm beam exceeds the titanium's fatigue endurance limit at 107 cycles, which is about 300 MPa, suggesting that fatigue failure is likely once the 25-mm cantilevers accumulate about 10 million cycles of loading at 300 N. On the other hand, because the peak stress in the 17-mm cantilever is only 222 MPa, fatigue is unlikely in that prosthesis.

The value in understanding key features of stress and failure is that successful prosthesis design can be accomplished via informed material selection and geometric design.

Appendix References

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3. Monje A, Ravidà A, Wang HL, Helms JA, Brunski JB. Relationship between primary/mechanical and secondary/biological implant stability. Int J Oral Maxillofac Implants 2019;34:s7–s23.